

B. Coulomb failure stress

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To better understand the physics behind the apparent correlation between seismicity and lake level, change of stress, particularly shear stress acts on pre-existing faults, due to dam impoundment must be estimated. On a given fault surface, ΔCFS is defined as:

$$\Delta CFS = \Delta \tau - \mu \Delta \sigma_e \quad (B-1)$$
$$\sigma_e = \sigma - p$$

where μ is the friction coefficient, $\Delta \tau$ and $\Delta \sigma_e$ are changes of shear stress and effective normal stress, respectively. Under undrained conditions (such as stress induced by a passing surface wave), the change in pore pressure due to a change in normal stress (where compression is positive) is given by $\Delta P = B \Delta \sigma$. Thus

$$\Delta CFS = \Delta \tau - \mu' \Delta \sigma, \mu' = (1 - B) \mu.$$

Following impoundment of a reservoir, both gravity loading and hydro pressure diffusion can lead to a significant change of stress on underlying and nearby faults.

B.1 Gravity loading

The deformation $S(x, y, z)$ caused by a distributed surface force $F(x, y)$ can be expressed as a convolution of Green's function $G(x, y, z)$ and the surface force. In the case of a surface water body, Green's function is the solution for Boussinesq's problem, which refers to a point force (Fv) vertically acting on the surface of a homogenous elastic half-space (see Jaeger and Cook, 1979; Liu and Zoback, 1992; Lei et al., 2008 for full sets of equations).

The assumption of homogeneous elastic half-space is sufficient for the purposes of this study. An example of deformation induced by reservoir filling is the impoundment of Hoover dam in 1935, which caused crustal blocks to sag downward as much as 12 cm, as observed by direct measurements. This deformation agrees well with theoretical calculations using Boussinesq's method (Carder, 1970).

B.2. Diffusion of pore pressure from injection and dam impoundment

Following Biot (1962), Rice and Cleary (1976), Bell and Nur (1978), the relationship governing pore pressure is given by the following diffusion equation:

$$\frac{1}{S_a} \nabla \cdot \left[\frac{k}{\eta} \nabla P \right] = \frac{\partial P}{\partial t} + \frac{B}{3} \frac{\partial \sigma_{ii}}{\partial t}$$

$$S_a = \phi(\beta_f + \beta_\phi) = \frac{2}{9GB^2} \frac{1 - \nu_u}{1 - \nu} \frac{(1 - \nu_u)(\nu_u - \nu)}{(1 + \nu_u)^2} \quad (\text{B-2})$$

where:

- P : Pore pressure, Pa ($\text{N} \cdot \text{m}^{-2}$, $\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2}$)
- S_a : Unconstrained specific storage coefficient, Pa
- k : Permeability, m^2
- η : Water viscosity, $\text{Pa} \cdot \text{s} = \text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$
- G : Rigidity or, in other words, shear modulus, Pa
- N : Poisson's ratio
- ν_u : Undrained Poisson's ratio
- B : Skempton's coefficient (ratio of pore pressure increment to mean stress increment under undrained conditions)
- Φ : Porosity, dimensionless
- β_f : Fluid compressibility, Pa^{-1}
- β_ϕ : Pore compressibility, Pa^{-1}

For an irrotational displacement field, equation (2) is mathematically uncoupled from the mechanical equilibrium equations, so pore pressure perturbations propagate independently of stress changes. If K and η are homogeneous, equation (2) can be further simplified as:

$$D \nabla^2 P = \frac{\partial P}{\partial t}, \quad D = \frac{k}{\eta S_a} \quad (\text{B-3})$$

The following are other formation properties often used in the related literature:

- D : Hydraulic diffusivity, $\text{m}^2 \cdot \text{s}^{-1}$
- K : Hydraulic conductivity, $\text{m} \cdot \text{s}^{-1}$
- S_s : Specific storage, m^{-1}
- A : Compressibility of matrix, $\alpha = \phi \beta_\phi$, $\text{Pa}^{-1} (\text{kg}^{-1} \cdot \text{m} \cdot \text{s}^2)$
- ρ : Density, $\text{kg} \cdot \text{m}^{-3}$

These parameters are linked by the following equations:

$$D = \frac{K}{S_s} \quad (B-4)$$

$$S_s = \rho g S_a = \rho g (\alpha + \phi \beta_f)$$

The change in pressure due to a water level change in a reservoir can be obtained by the numerical solution of equation (2) or (3), given initial and boundary conditions that correspond to the history of the impoundment of the reservoir.