Elastic wave in porous media simulated using MacCormack differential scheme

Contents

1 Introduction

The wave propagation in 3-D heterogeneous poroelastic media is simulated using the MacCormack differential scheme which is extended from 2D case (Dai et al., 1995).

1.1 Biot's wave equation

Biot's wave equation are

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{A}\frac{\partial \mathbf{u}}{\partial x} + \mathbf{B}\frac{\partial \mathbf{u}}{\partial y} + \mathbf{C}\frac{\partial \mathbf{u}}{\partial z} + \mathbf{D}\mathbf{u}$$

$$\mathbf{u} = \begin{pmatrix} v_x & v_y & v_z & \sigma_{xx} & \sigma_{yy} & \sigma_{zz} & \tau_{yz} & \tau_{zx} & \tau_{xy} & q_x & q_y & q_z & p \end{pmatrix}^T$$

where,

v_x, v_y, v_z:	solid velocity
q_x, q_y, q_z:	fluid velocity
σ,τ:	components of stress tensor
p:	Pore pressure

1.2 The finite-difference scheme

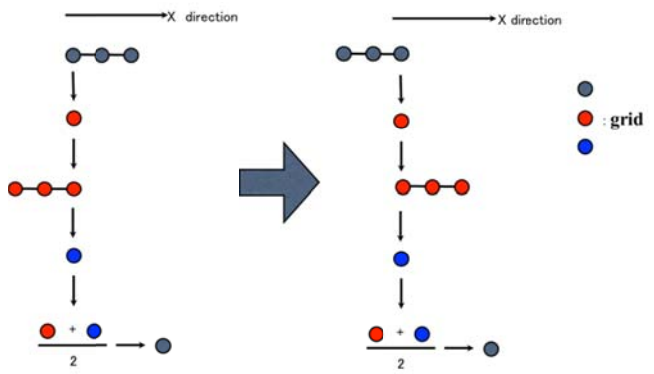


Fig. 1 Schematic diagram of the MacCormack differential scheme

Step-1:

$$\mathbf{u}^* = \mathbf{u}^0 - \frac{\Delta t}{6\Delta x}\mathbf{A}\left(7\mathbf{u}^1 - 8\mathbf{u}^2 + \mathbf{u}^3\right) \text{ --> } \mathbf{u} = \frac{1}{2}\left(\mathbf{u}^0 + \mathbf{u}^*\right) + \frac{\Delta t}{12\Delta x}\mathbf{A}\left(7\mathbf{u}^{-1} - 8\mathbf{u}^{-2} + \mathbf{u}^{-3}\right)$$

Step-2:

$$\mathbf{u} = \frac{1}{2}\left(\mathbf{u}^0 + \mathbf{u}^*\right) - \frac{\Delta t}{12\Delta x}\mathbf{A}\left(7\mathbf{u}^1 - 8\mathbf{u}^2 + \mathbf{u}^3\right) \text{ --> } \mathbf{u}^* = \mathbf{u}^0 + \frac{\Delta t}{6\Delta x}\mathbf{A}\left(7\mathbf{u}^{-1} - 8\mathbf{u}^{-2} + \mathbf{u}^{-3}\right)$$

2 Input file format

Model parameters are defined in a <XML> style text file. A valid input file has an extension of ".mac". For 2D model, the word "2D" must be included within the file name, such as "model_par_2D.mac".

```
<?xml version="1.0" encoding="Shift_JIS" ?>
<GeoTaos lib="MAC">
   <memo>
      <titlet>Demo model for Mac</title>
      <kongxier>Xinglin Lei, 2016/02/10</kongxier>
   </memo>
   <scalling>
      <unit>l="m", t="sec"</unit>
   </scalling>
   <mesh type="halfspace">           //*1
      <x>min=0, max=300, ng=301</x>
      <y>min=0, max=150, ng=151</y>
      <z>min=0, max=150, ng=151</z>
      <t>dt=0.0002, nt=2000, nt_0=0</t>        //*2
      <snap>ns=0, ne=1000, n_jump=5</snap>
      <opt>sigma=0.1</opt>
   </mesh>
   <source>
      <src_fun>type=6, amp=1.0, t0=0.0, fc=80</src_fun>
      <src_i type="iso">x=50, y=75, z=80, pot=1000.0, r=2<src_i> //r=1: <-|-> 2:<-<-|->->, so on
   </source>
   <station>
      <sta_>p0= 10, 75, 2, p1=290, 75,   2, delta=2</sta_>
      <sta_>p0=250, 75, 4, p1=250, 75, 150, delta=2</sta_>
   </station>
   <formation>
    <stra ind=0, "shale">Ks=35.00e9, den=2600.0, Kd=8.00e9, G=7.50e9, Fai=0.05, perm=1.0e-12, Toet=3.0, Kf=2.51e9, den_f=1040, visc=0.5e-3</stra>
    <stra ind=1, "snd_w">Ks=35.00e9, den=2500.0, Kd=6.00e9, G=6.00e9, Fai=0.15, perm=1.0e-10, Toet=3.0, Kf=2.51e8, den_f=1040, visc=0.5e-3</stra>
    <stra ind=2, "snd_g">Ks=35.00e9, den=2450.0, Kd=6.00e9, G=6.00e9, Fai=0.15, perm=1.0e-10, Toet=3.0, Kf=1.44e6, den_f=100,  visc=2.2e-5</stra>
       rock=0, Z:  0   150
       rock=1, Z:  100 150
       rock=2, XYZ: 75 225 0 150  100 120
   </formation>
</GeoTaos>
```

*1 type="cube" for the case of reflective boundaries, otherwise, except the upper boundary, all other boundaries are adsorbing.

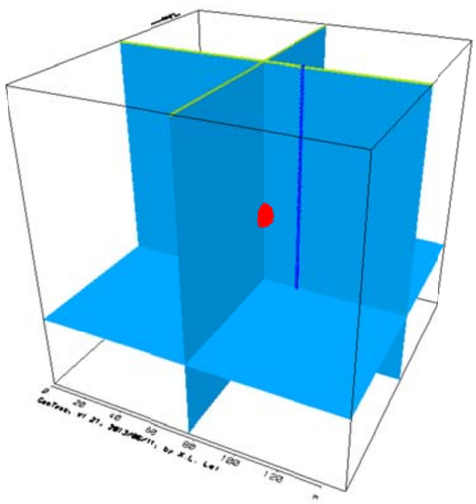
*2 nt_0: used for consecutive running. At each running the final solution would be saved into a binary file with double precision and a file name as "#####_mac.sav", where ##### indicate the final number of time steps of previous run. By setting nt_0 equal to the number, a consecutive run can be done without any additional error.

3. Treatment at boundaries

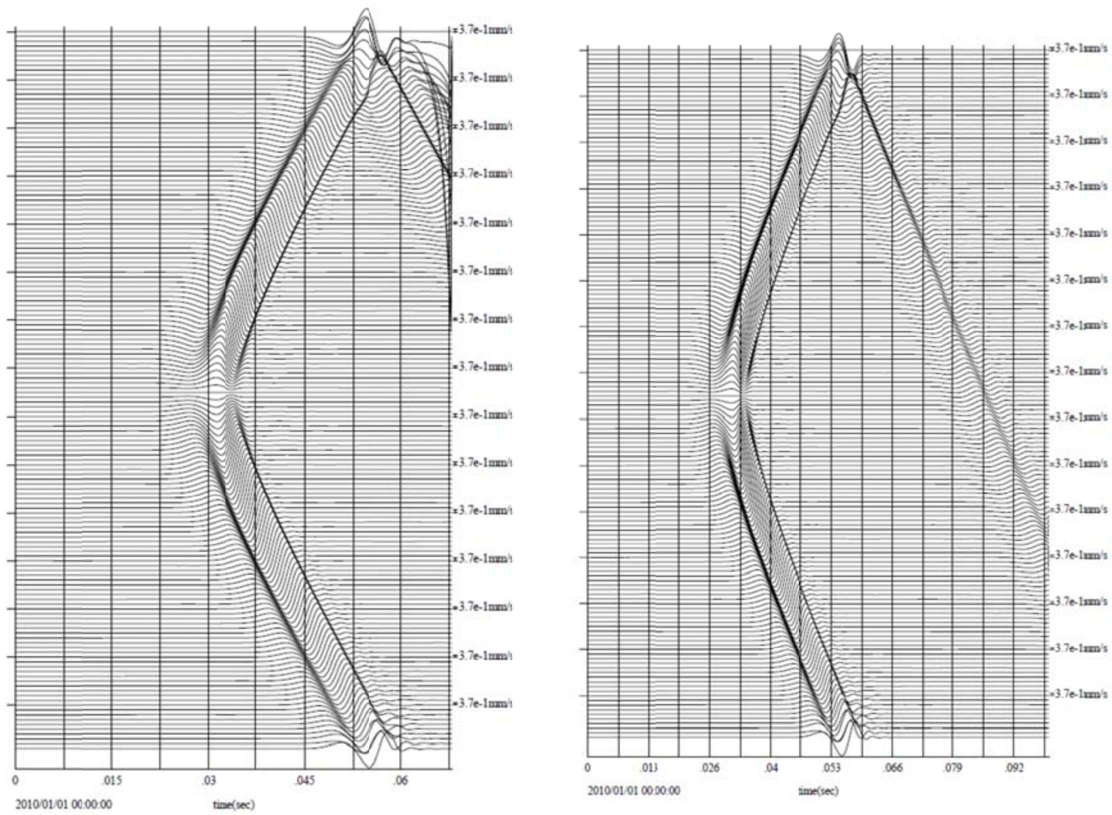
For reflect boundary at the ground surface, the original codes do not convergence when the maximum permeability of the model is greater than 10^{-5} (the original codes ignore the drag force by setting b=0). This problem is overcome by some corrections, as shown in red fonts in the following list.

```
k = zs + 1;                                                 // Upper boundary 地表面の境界条件
for ( j = ys+1; j <= ye-1; j++ ) {
     for ( i = xs+1; i <= xe-1; i++ ) {
              VZS[k][j][i] =  VZS[k+2][j][i];
              TZZ[k][j][i] = -TZZ[k+2][j][i];
              VZF[k][j][i] =  VZF[k+2][j][i];
               SS[k][j][i] =  -SS[k+2][j][i];
              //XL Lei 2016/2/16
              VZS[k+1][j][i] = VZS[k+2][j][i];
              VZF[k+1][j][i] = VZF[k+2][j][i];
              TZZ[k+1][j][i] = 0;
              SS[k+1][j][i]  = 0;
     }
}
```

3.2 An example



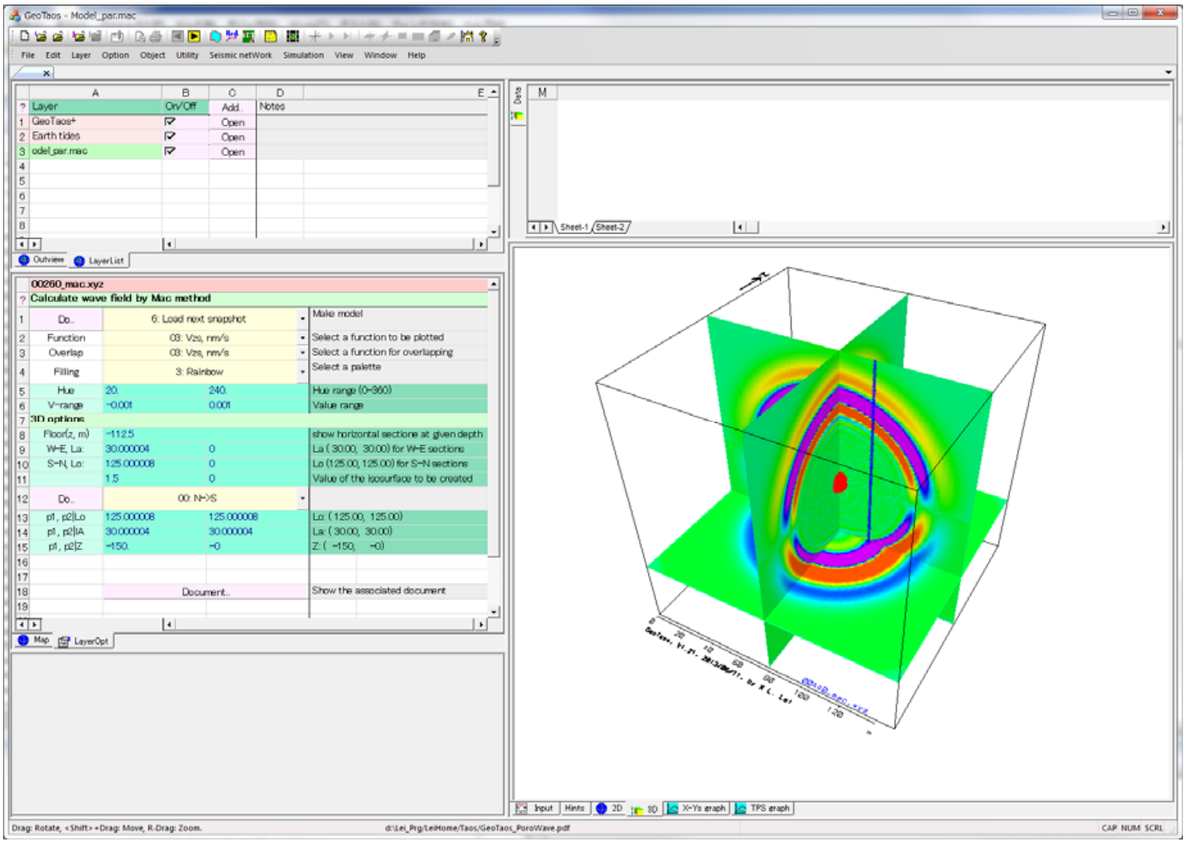
A homogeneous model for testing.



As shown in the left plot in Fig. 3, the original codes do not work when wave front reached the ground surface and begin to reflect. While the corrected codes work well.

4. Integrated interface

An XL_GmapMac and XL_GmapMac2D layer were developed and added into the GeoTaos framework to provide integrated interface for numerical simulation of wave propagation in porous media using the MacCormack finite difference scheme. A valid input file has an extension of ".mac". For 2D model, the word "2D" must be included within the file name, such as "model_par_2D.mac".



The XL_GmapMac layer—integrated interface for simulating seismic wave propagation in porous media by use of the MacCormack method.

4.1 Run simulation from GeoTaos

1) Start GeoTaos, drag and drop a valid input file, ???.mac, in to the client area of the GeoTaos window.

2) Click the <LayerList> and the target item to show the LayerOpt of the GmapMac layer.

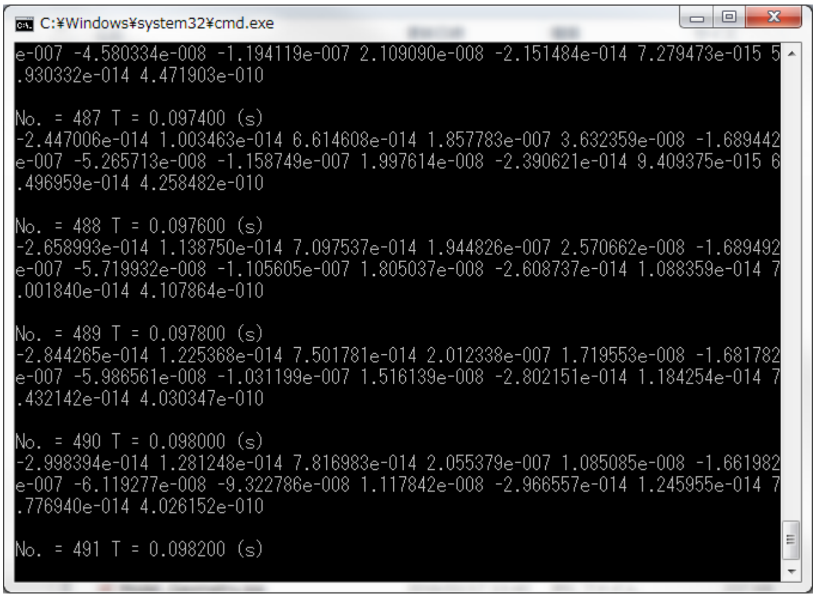
3) Chose "Start" from the list of Row-1, and click <Do.> of the same row. 25 time steps would be executed.

4) Select function to be shown from Row-2, properly change the range of value from Row-6, and other parameters of visualization.

5) To continue simulation, chose "Forward" from Row-1 and click <Do..>. Simulation would be automatically forwarded until the time steps reached the final number given in the input file or the "End" is selected from Row-1.

4.2 Run the CONSOL version

Type "Mac Model_par.max Model_out.dat" from the DOS window. Time steps and some limited data are streamed to the window.



4.2 Convert simulated waveform data to SAC files

1) Be sure the simulation is completed and the output file "Model_out.dat" has been created.

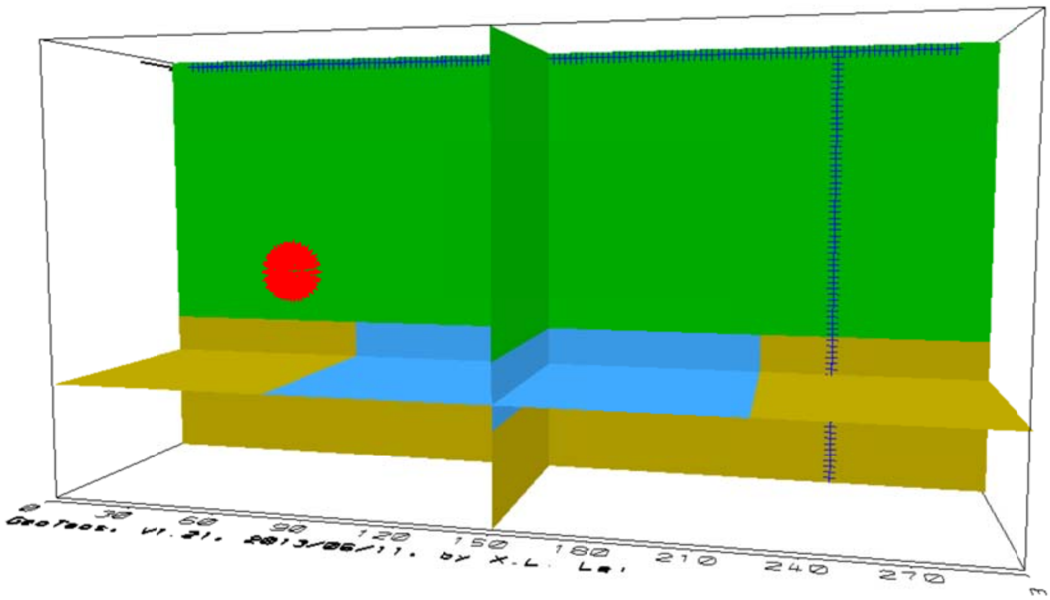
2) Create a folder for SAC files.

3) Chose "Results->SACs" from the list of Row-1, and click <Do.>. Select the fold has been created. Then, SAC files for every component and station would be created in the specified fold.

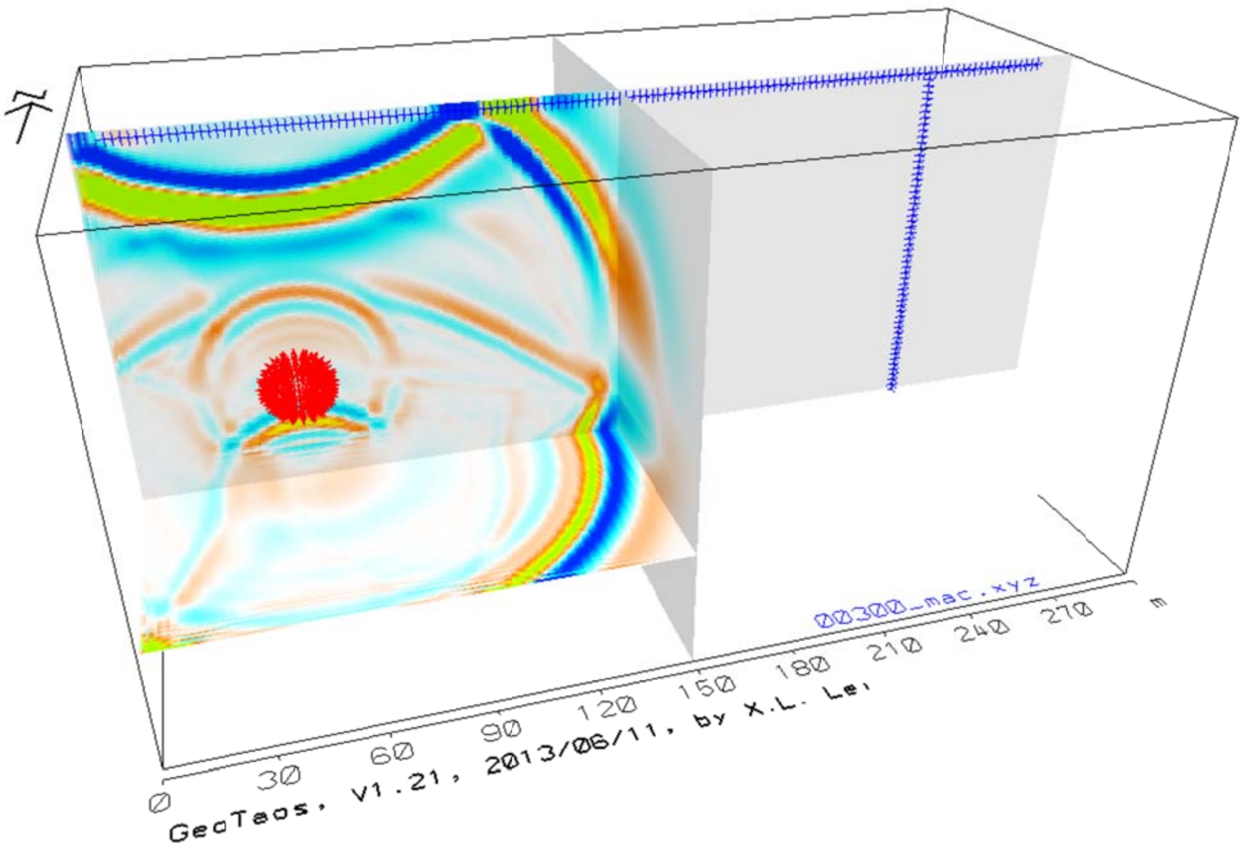
5. A 3D reservoir example

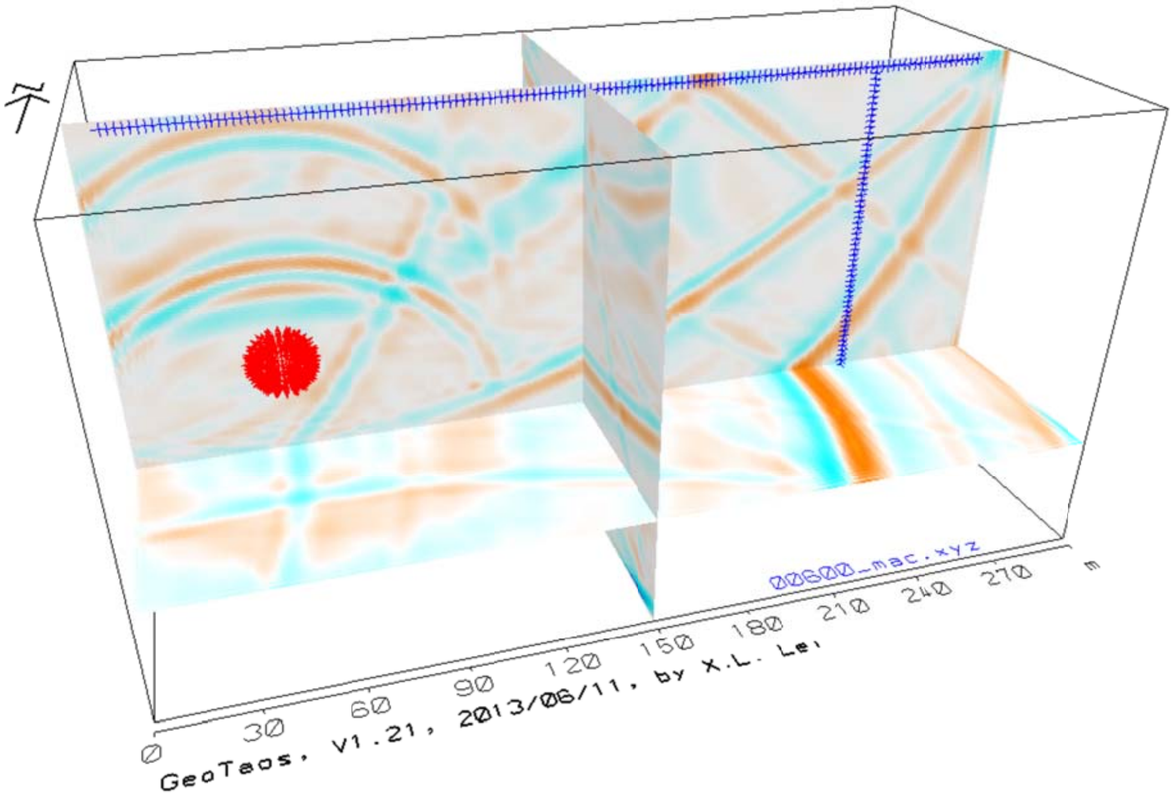
5.1 Model

```
<mesh>
   <x>min=0, max=300,  ng=301</x>
   <y>min=0, max=150,  ng=151</y>
   <z>min=0, max=150,  ng=151</z>
   <t>dt=0.0002, nt=2000, nt_0=0</t>
   <snap>ns=0, ne=1000, n_jump=5</snap>
   <opt>sigma=0.1</opt>
</mesh>

<source>
   <src_fun>type=6, amp=1.0, t0=0.024, fc=80</src_fun>
   <src_i type="iso">x=50, y=75, z=80, pot=1000.0, r=6<src_i>
</source>

<station>
   <sta_>p0= 10, 75, 2, p1=290, 75,   2, delta=2</sta_>
   <sta_>p0=250, 75, 4, p1=250, 75, 150, delta=2</sta_>
</station>

<formation>
   <stra ind=0, "shale">Ks=9.00e9, den=2600.0, Kd=8.00e9, G=7.50e9, Fai=0.05, perm=1.0e-12, Toet=3.0, Kf=2.51e9, den_f=1040, visc=0.5e-3</stra>
   <stra ind=1, "snd_w">Ks=8.00e9, den=2500.0, Kd=6.00e9, G=6.00e9, Fai=0.15, perm=1.0e-10, Toet=3.0, Kf=2.51e8, den_f=1040, visc=0.5e-3</stra>
   <stra ind=2, "snd_g">Ks=8.00e9, den=2450.0, Kd=6.00e9, G=6.00e9, Fai=0.15, perm=1.0e-10, Toet=3.0, Kf=1.44e6, den_f=100,  visc=2.2e-5</stra>
    rock=0, Z:  0   150
    rock=1, Z:  100 150
    rock=2, XYZ: 75 225 0 150  100 120
</formation>
```

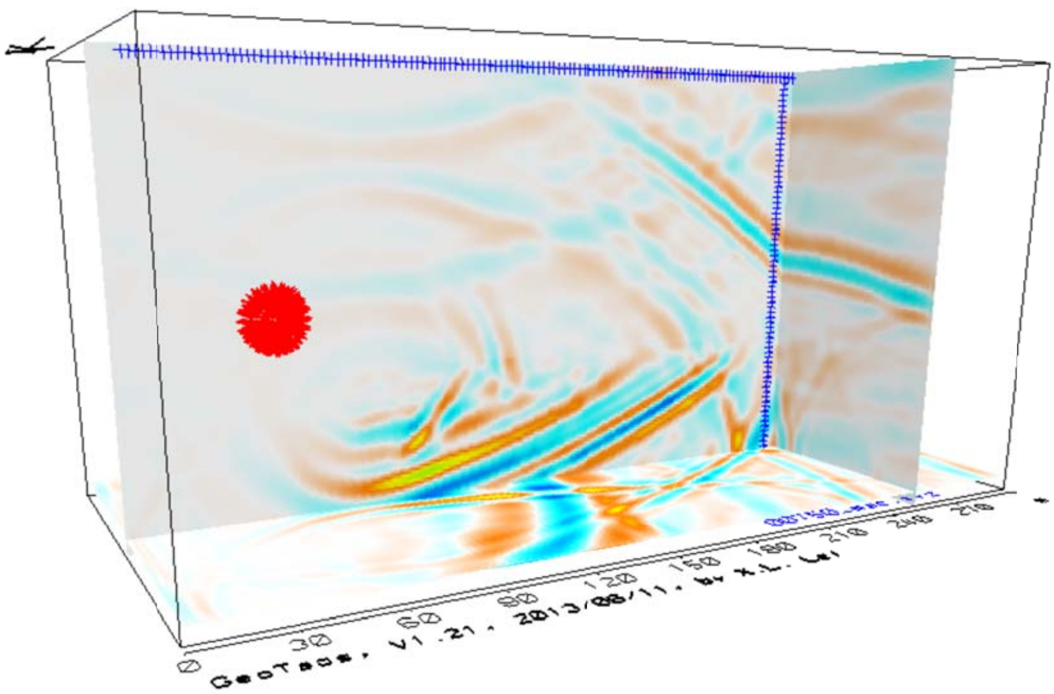


Configurations of the test reservoir, locations of the explosive source and observation stations





Snapshots of the z-component solid velocity at 600 and 1200 ms.



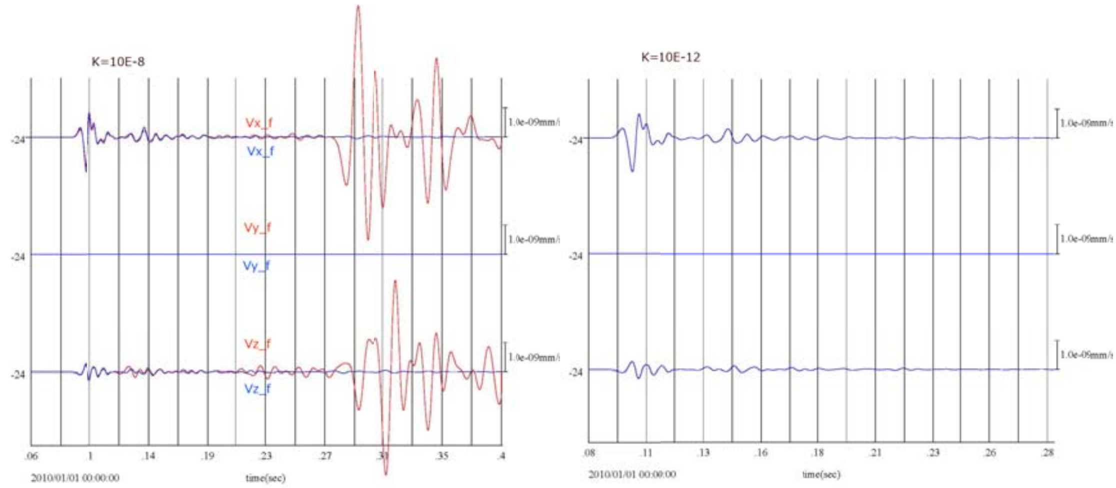
Difference wave field showing the effect of the gas layer

5.2 Sensitivity on permeability

The permeability of each layer is changed to a smaller value as follows.

```
<formation>
      <stra ind=0, ″shale″>..., perm=1.0e-8, ...</stra>
      <stra ind=1, ″snd_w″>..., perm=1.0e-6, ...</stra>
      <stra ind=2, ″snd_g″>..., perm=1.0e-6, ...</stra>
```

Following plots show a comparison of particle velocities recorded at (250, 75, 2**4**). There are significant differences between fluid velocity and solid velocity in results of the second model having greater permeability.



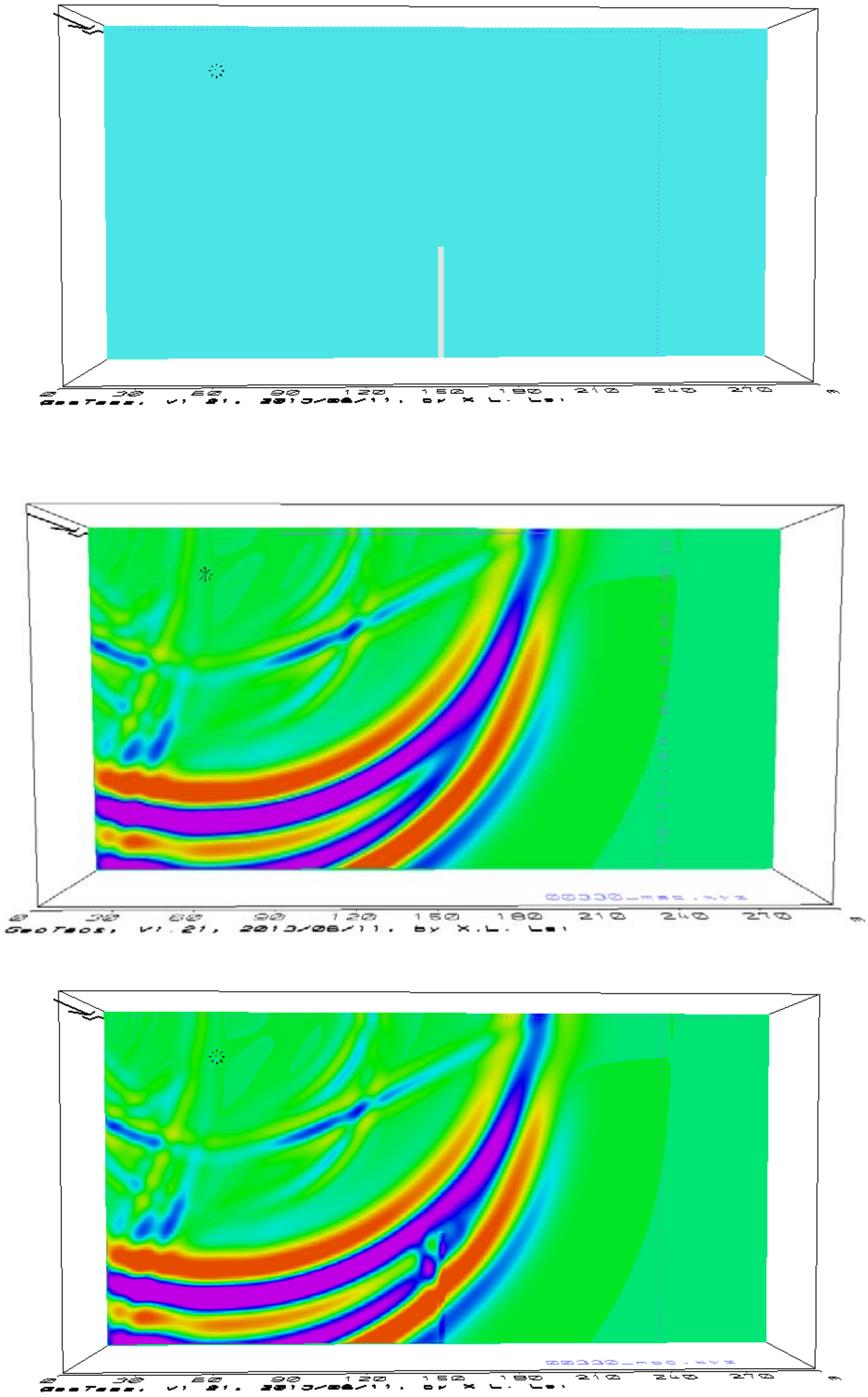
6. A 2D example

```
   <mesh>
      <x>min=0, max=300, ng=601</x>
      <y>min=0, max=100, ng=2</y>
      <z>min=0, max=150, ng=301</z>
      <t>dt=0.0002, nt=1000, nt_0=0</t>
      <snap>ns=0, ne=1000, n_jump=10</snap>
      <opt>sigma=0.1</opt>
   </mesh>

   <source>
      <src_fun>type=6, amp=2.0, t0=0.0, fc=100</src_fun>
      <src_i type="iso">x=50, y=0, z=20, pot=1000.0, r=2<src_i>
   </source>

   <station>
      <sta_>p0= 10, 0, 2, p1=290, 0,   2, delta=2</sta_>
      <sta_>p0=250, 0, 4, p1=250, 0, 150, delta=2</sta_>
   </station>

   <formation>
      <stra ind=0, "shale">Ks=30.0e9, den=2600.0, Kd=8.00e9, G=7.50e9, Fai=0.05, perm=1.0e-12, Toet=3.0,
Kf=2.51e9, den_f=1040, visc=0.5e-3</stra>
       rock=0, Z:  0   150
   </formation>
```

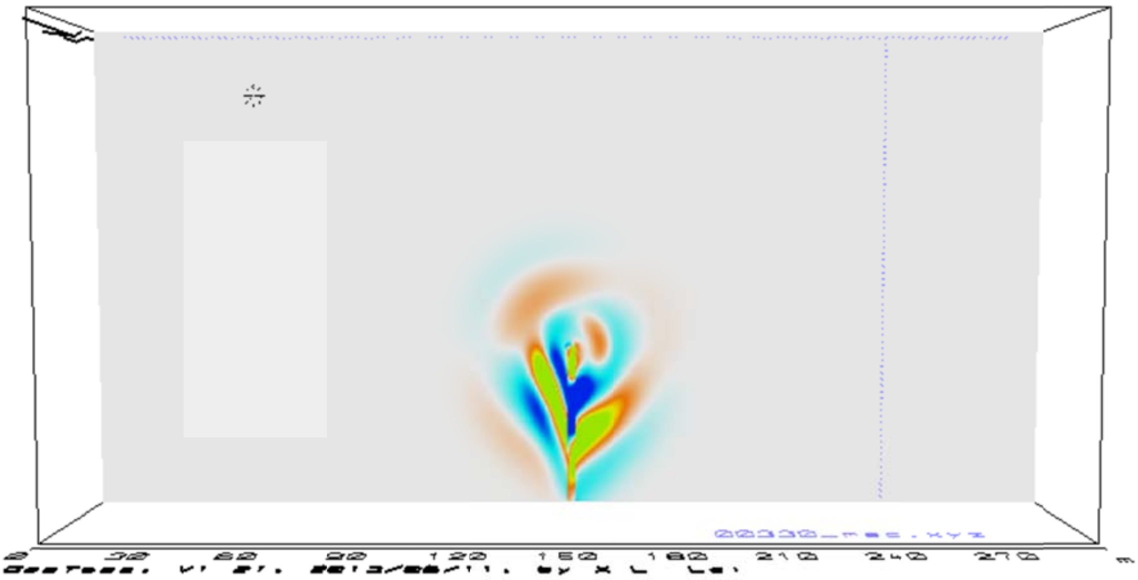
A homogeneous 2D model.

```
  <formation>
      <stra ind=0, "shale">Ks=30.0e9, den=2600.0, Kd=8.00e9, G=7.50e9, Fai=0.05, perm=1.0e-12, Toet=3.0,
Kf=2.51e9, den_f=1040, visc=0.5e-3</stra>
      <stra ind=1, "fault">Ks=20.0e9, den=2500.0, Kd=6.00e9, G=6.00e9, Fai=0.15, perm=1.0e-08, Toet=3.0,
Kf=2.51e8, den_f=1040, visc=0.5e-3</stra>
       rock=0, Z:  0   150
       rock=1, XZ: 150 152 100 150
   </formation>
```

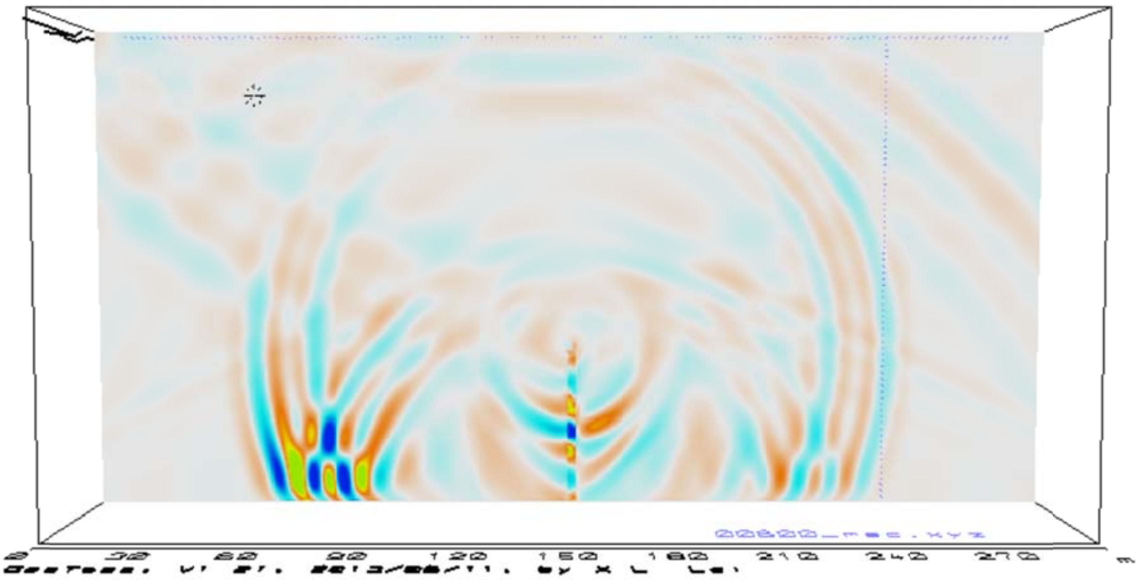
In addition, with a vertical fault.



A snapshot of vz_s at 660 ms for the homogeneous model (upper) and fault model (lower)

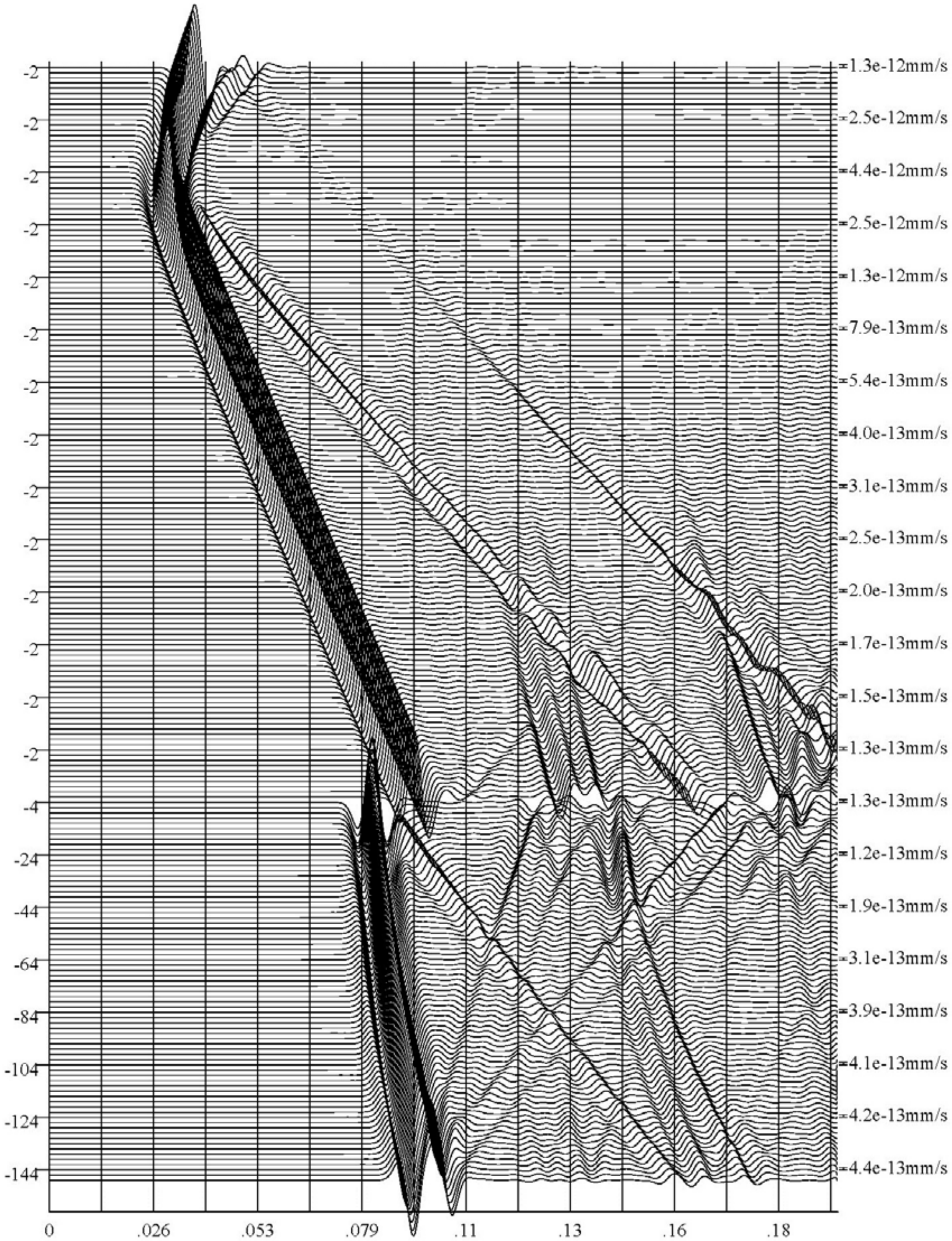


t=660 ms



t=1.2 s

Difference wave field showing fault scattering



Vz_s at given station along a surface line and a borehole

Reference

Dai, N., Vafidis, A., Kanasewich, E., 1995. Wave propagation in heterogeneous, porous media: A velocity-stress, finite-difference method. Geophysics 60, 327-340.