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Determination of Aquifer Transmissivity From Earth Tide Analysis

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The water level in an open well tapping an artesian aquifer responds to pressure head disturbance caused by Earth tide dilatation of the aquifer. Because a finite amount of time is needed for water to flow into and out of the well, there exists a phase shift (or time lag) between the tidal dilatation of the aquifer and the water level response in the well. We derive an analytical solution that expresses the phase shift as a function of the aquifer transmissivity, storage coefficient, well radius, and the period of the harmonic disturbance. This solution is rather insensitive to the storage coefficient. Thus if the phase shift is known for a harmonic disturbance, the transmissivity can be calculated given a rough estimate of the storage coefficient. Theoretical analysis shows that a significant phase shift may be present even if the disturbance is slowly varying, as in the case of Earth tides. This opens the possibility of estimating aquifer transmissivity from water level records that show Earth tide fluctuations. A case study, using data from a site near Parkfield, California, is presented to illustrate application of the theory. Phase shifts of the O_1 (25.82-hour period) and M_2 (12.42-hour period) tidal constituents are chosen for analysis because they are free of systematic contamination by fluctuations in barometric pressure. A brief error analysis suggests that the computed O_1 phase shift is subject to large uncertainty, while the computed M_2 phase shift is substantially more accurate. Based on an assumed storage coefficient range of 10^{-4} to 10^{-6}, the estimated transmissivity range is 8×10^{-6} to 2×10^{-5} m²/s. While hydraulic tests have not been performed to validate these estimates, the range is consistent with the transmissivity value determined by other investigators from analysis of the water level response to an earthquake.

INTRODUCTION

It is well-known that the water level in an open well tapping an artesian aquifer responds to pressure head disturbances caused by dilation of the aquifer. Of particular interest are harmonic disturbances, which may be caused by seismic waves or Earth tides. *Cooper et al.* [1965] showed that the steady fluctuation of water level in a well occurs at the same frequency as the harmonic pressure head disturbance in the aquifer. However, the amplitude of the response is generally different from that of the disturbance, and there is also a shift in phase. We can describe the pressure head disturbance and the water level response by

$$h_f = h_0 \exp(i\omega t) \tag{1}$$

$$x = x_0 \exp(i\omega t) \tag{2}$$

respectively, where

h_f fluctuating pressure head in the aquifer, L;
h_0 complex amplitude of pressure head fluctuation, L;
x displacement of water level from static position, L;
x_0 complex amplitude of water level fluctuation, L;
$i = (-1)^{1/2}$;
t time, T;
$\omega = 2\pi/\tau$ = frequency of fluctuation, T^{-1};
τ period of fluctuation, T.

Complex notation is used in (1) and (2) to facilitate the theoretical development below. However, it is understood that we are interested in only the real parts of h_f and x. The amplitude response A is defined as the ratio between the amplitude of the water level fluctuation and that of the pressure head fluctuation. In terms of x_0 and h_0, A can be expressed as

$$A = |x_0/h_0| \tag{3}$$

The phase shift is defined as

$$\eta = \arg(x_0/h_0) \tag{4}$$

where arg (z) is the argument of the complex number z. The phase shift can be thought of as being $2\pi t_p/\tau$, where t_p is the time lag from the moment when the water level fluctuation reaches a peak to the moment when the pressure head fluctuation reaches a peak. Note that the phase shift is negative when the water level response lags behind the pressure head disturbance.

Analysis of water level fluctuations have generally focused on the amplitude response. *Cooper et al.* [1965] showed that the amplitude response depends on (1) the aquifer properties, i.e., the transmissivity T and storage coefficient S, (2) the radius of the well casing r_c, (3) the period of the pressure head disturbance τ, and (4) the inertial effects of water in the well. As an example, consider an artesian well aquifer system where $S = 10^{-4}$ and $T/r_c^2 = 1.0$ s^{-1}. For pressure disturbances with very short periods (less than 0.5 s) the water level in the well cannot respond rapidly enough to keep up with the pressure disturbance in the aquifer, and the well acts as a low-pass filter, tending not to respond ($A \approx 0$). For periods of 2–30 s (typical of seismic Reyleigh waves) it is possible for the well to be excited in a sympathetic manner so as to amplify the magnitude of the disturbance ($A > 1$). At very long periods, the well behaves more or less as a manometer, and the water level change is approximately equal to the pressure head change in the aquifer ($A \approx 1$). In rather simple terms, an artesian well

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aquifer system can be viewed as a forced harmonic oscillator with viscous damping, the degree of over- or underdamping depending upon the inertial effects and the ease with which water can move into and out of the well.

For Earth tide analysis (with semidiurnal and diurnal periods) it is generally assumed that inertial effects are negligible and that the amplitude response approaches one. For these conditions, *Bredehoeft* [1967] presented a method for determining the specific storage of the aquifer material if Poisson's ratio of the aquifer material is known. Bredehoeft's analysis was recently criticized by *Narasimhan et al.* [1984] as being inconsistent. While we do not agree with the criticisms of Narasimhan et al., we recognize that these criticisms do not bear directly on the material presented in the present paper. Therefore a detailed discussion of Brehedoeft's analysis is deferred to a separate paper (P. A. Hsieh et al., Response of well aquifer systems to Earth tide: A reexamination, submitted to *Water Resources Research*, 1987).

Several investigators have studied the phase difference between tidal disturbance and water level fluctuations. Both *Hanson and Owen* [1982] and *Bower* [1983] dealt with the case of a well intersecting a single fracture. The phase shift was related to the orientation of the fracture plane. *Gieske and De Vries* [1985] analyzed the case of a compressible aquifer surrounded by relatively rigid aquifers. They attributed the phase shift to groundwater flow induced by tidal strain differential between the adjacent aquifers. In the present study, we consider a single, laterally extensive, confined aquifer that is homogeneous and isotropic. The phase shift is assumed to be caused by the time needed for water to flow into and out of the well (i.e., well bore storage effects). This type of phase shift has not been well studied in the past. It was generally assumed that as the amplitude response approaches one, the disturbance and the response would be in phase. While this is true at very long periods, there are periods for which the amplitude response is approximately one and yet a significant phase shift exists. *Narasimhan et al.* [1984] recognized this possibility and presented results of numerical simulations to illustrate the effects of well bore storage, period of disturbance, and aquifer properties, on the amplitude response and the phase shift. They briefly outlined an approach, based on type curve matching, for estimating aquifer transmissivity from knowledge of the amplitude responses and phase shifts associated with a number of tidal constitutients [see *Narasimhan et al.*, 1984, p. 1919]. However, the usefulness of this method has not been demonstrated.

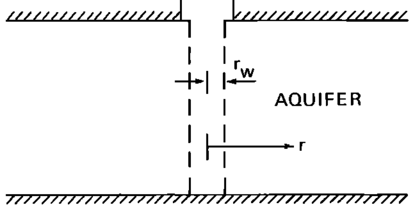

Fig. 1. Idealized representation of open well drilled into a confined aquifer.

In the present paper we examine the phase shift by an analytical procedure that is similar to that of *Cooper et al.* [1965], but we neglect inertial effects of water in the well bore. We develop a method for estimating aquifer transmissivity from the phase shift associated with each tidal constitutient. A case study is presented to illustrate application of the method. The aim of our approach is not to replace hydraulic testing. What we wish to point out is that water level data collected under natural conditions may contain a great deal of useful information. For example, it is not unusual that a large-scale hydraulic testing program is preceded by a period of baseline monitoring. If tidal effects are observed during monitoring, then the phase analysis may provide valuable information for design of the hydraulic tests.

THEORY

Figure 1 shows an artesian well aquifer system similar to the one considered by *Cooper et al.* [1965]. The well is open, nonflowing, and it penetrates the entire thickness of the confined aquifer, which is assumed to be isotropic, homogeneous, and of large lateral extent. The radius of the screened or open portion of the well is denoted by r_w. The movement of the water level is assumed to occur within the well casing, which has radius r_c.

The pressure head disturbance in the aquifer produces a discharge from the aquifer to the well. This discharge is

$$Q = \pi r_c^2 \frac{dx}{dt} = i\omega x_0 \pi r_c^2 \exp(i\omega t) \tag{5}$$

Cooper et al. [1965] writes "This discharge produces in the aquifer a drawdown s (positive downward) which is superimposed on the fluctuating pressure." In the absence of inertial effects, the fluctuating water level and the pressure head are related by

$$x = h_f - s_w \tag{6}$$

where s_w is the drawdown at the well due to the discharge Q.

In Appendix A we derive a solution for the drawdown in a well with a periodic discharge at a volumetric rate of $Q_0 \exp(i\omega t)$. This solution is

$$s_w = \frac{Q_0}{2\pi T} \{[\Phi \operatorname{Ker}(\alpha_w) - \Psi \operatorname{Kei}(\alpha_w)] + i[\Psi \operatorname{Ker}(\alpha_w) + \Phi \operatorname{Kei}(\alpha_w)]\} \exp(i\omega t) \tag{7}$$

where

$$\Phi = \frac{-[\operatorname{Ker}_1(\alpha_w) + \operatorname{Kei}_1(\alpha_w)]}{2^{1/2}\alpha_w[\operatorname{Ker}_1^2(\alpha_w) + \operatorname{Kei}_1(\alpha_w)]} \tag{8}$$

$$\Psi = \frac{-[\operatorname{Ker}_1(\alpha_w) - \operatorname{Kei}_1(\alpha_w)]}{2^{1/2}\alpha_w[\operatorname{Ker}_1^2(\alpha_w) + \operatorname{Kei}_1^2(\alpha_w)]} \tag{9}$$

$$\alpha_w = \left(\frac{\omega S}{T}\right)^{1/2} r_w \tag{10}$$

with Ker (α_w) and Kei (α_w) being Kelvin functions of order zero, and $\operatorname{Ker}_1(\alpha_w)$ and $\operatorname{Kei}_1(\alpha_w)$ being Kelvin functions of order one. According to (5), Q_0 should be $i\omega x_0 \pi r_c^2$ and (7) becomes

$$s_w = -\frac{\omega r_c{}^2 x_0}{2T}\{[\psi \text{ Ker }(\alpha_w) + \Phi \text{ Kei }(\alpha_w)] - i[\Phi \text{ Ker }(\alpha_w) - \Psi \text{ Kei }(\alpha_w)]\} \exp(i\omega t) \quad (11)$$

Substituting (1), (2), and (11), into (6), dividing through by exp ($i\omega t$), and rearranging we obtain

$$x_0/h_0 = (E + iF)^{-1} \quad (12)$$

where E and F are defined as

$$E = 1 - \frac{\omega r_c{}^2}{2T}[\Psi \text{ Ker }(\alpha_w) + \Phi \text{ Kei }(\alpha_w)] \quad (13)$$

$$F = \frac{\omega r_c{}^2}{2T}[\Phi \text{ Ker }(\alpha_w) - \Psi \text{ Kei }(\alpha_w)] \quad (14)$$

According to (3) and (4), the amplitude response is

$$A = (E^2 + F^2)^{-1/2} \quad (15)$$

and the phase shift is

$$\eta = -\tan^{-1}(F/E) \quad (16)$$

For Earth tide analysis and for realistic values of r_w, T, and S, the value of α_w as computed by (10) will usually be small (<0.1). In this case, both $\text{Ker}_1(\alpha_w)$ and $\text{Kei}_1(\alpha_w)$ can be approximated by $-1/(2^{1/2}\alpha_w)$. This leads to $\Phi \approx 1$ and $\psi \approx 0$, and E and F can be approximated by

$$E \approx 1 - \frac{\omega r_c{}^2}{2T}\text{ Kei }(\alpha_w) \quad (17)$$

$$F \approx \frac{\omega r_c{}^2}{2T}\text{ Ker }(\alpha_w) \quad (18)$$

APPLICATION

The preceding analysis shows that the amplitude response A and the phase shift η are both functions of two dimensionless parameters: $\omega r_c{}^2/T$ and α_w. For groundwater applications, it is more convenient to use an alternative set of dimensionless parameters, $T\tau/r_c{}^2$ and $Sr_w{}^2/r_c{}^2$. Figures 2 and 3 show plots of η and A, respectively, versus $T\tau/r_c{}^2$ for various values of $Sr_w{}^2/r_c{}^2$. For Earth tide fluctuations with approximately semidiurnal and diurnal periods ($\tau \approx 12$ and 24 hours) and a well aquifer system for which $T/r_c{}^2$ varies between 10^{-5} to 10^{-2} s^{-1}, the value of $T\tau/r_c{}^2$ ranges from 1 to 1000. Figure 2 shows that significant phase shifts can occur. Furthermore, comparison between Figures 2 and 3 illustrates that where the amplitude response is 0.95 ($T\tau/r_c{}^2 \approx 10^2$), there still exists a phase shift of approximately 12°. Indeed, one has to look to amplitude responses approaching 0.99 ($T\tau/r_c{}^2 \approx 10^3$) before the phase shift is less than 2°. At amplitude responses of about 0.1, the phase shift can be quite large, approaching 80°

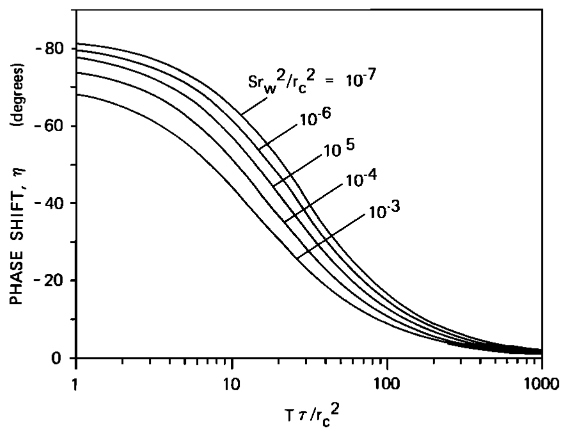

Fig. 2. Plot of phase shift η versus $T\tau/r_c{}^2$ for various values of $Sr_w{}^2/r_c{}^2$.

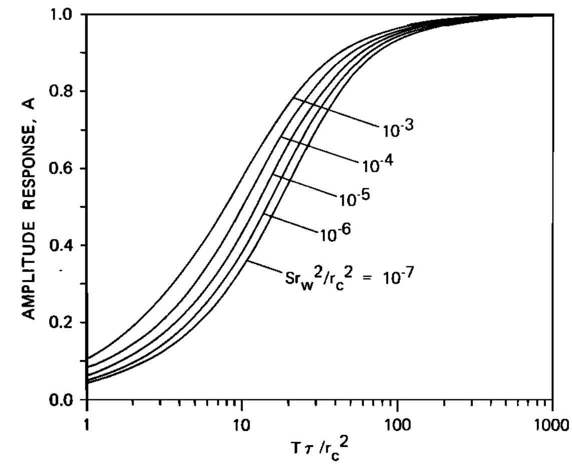

Fig. 3. Plot of amplitude response A versus $T\tau/r_c{}^2$ for various values of $Sr_w{}^2/r_c{}^2$.

Although Figure 2 contains all of the relevant information in terms of the phase shift η and the dimensionless variable $T\tau/r_c{}^2$, it does not illustrate clearly the relationship between the well aquifer parameter $T/r_c{}^2$ and the period of the disturbance τ. For this reason, we constructed Figure 4, which shows the phase shift as a function of τ for $Sr_w{}^2/r_c{}^2 = 10^4$ and various values of $T/r_c{}^2$. If we consider only harmonic disturbances with periods of about 12 and 24 hours, then for $Sr_w{}^2/r_c{}^2 = 10^{-4}$, Figure 4 indicates that phase lags are measurable for $T/r_c{}^2 < 10^{-2}$ s^{-1}. However, if $T/r_c{}^2 < 10^{-5}$ s^{-1} (so that $T\tau/r_c{}^2$ is less than approximately one), the magnitude of the water level fluctuation may be sufficiently dampened that it may not be measurable.

Figure 2 also shows that the relationship between η and $T\tau/r_c{}^2$ is not highly sensitive to changes in $Sr_w{}^2/r_c{}^2$. For a given η, varying $Sr_w{}^2/r_c{}^2$ from 10^{-3} to 10^{-7} (four orders of magnitude) changes $T\tau/r_c{}^2$ by only a factor of 2–4. Thus if the phase shift can be determined for a harmonic disturbance, Figure 2 can be used to estimate a range of values of $T\tau/r_c{}^2$ by assuming a likely range of values of $Sr_w{}^2/r_c{}^2$. Since τ and r_c are known, a range of T values can be calculated. If there exists an order-of-magnitude estimate of S (e.g., obtained from the analysis of *Bredehoeft* [1967]), then the uncertainty of T would be greatly decreased.

For Earth tide analysis, two sets of data are needed to determine the phase shift: (1) water level fluctuation in the well and (2) pressure disturbance in the aquifer. Because our analysis concerns phase relationships, the magnitude of the fluctuation in each record needs to be determined only to within a multiplicative constant. If near-continuous measurements (e.g., at hourly intervals or less) are made over a period of several months and if the signal-to-noise ratio is sufficiently small, then the phase shift can be determined to within a few degrees.

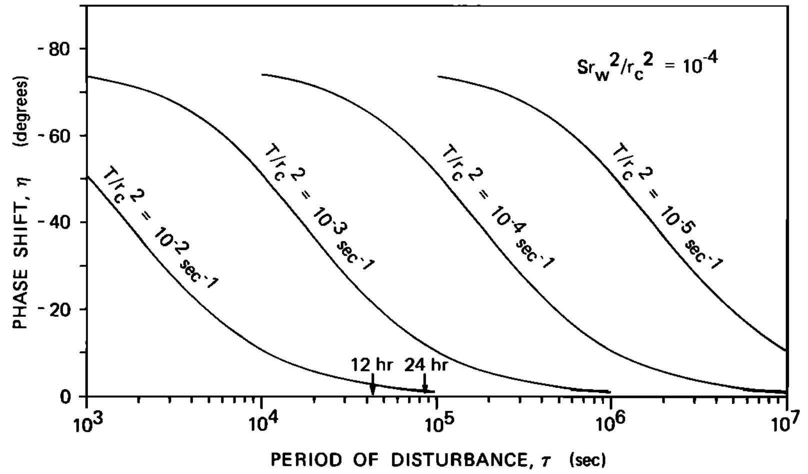

Fig. 4. Plot of phase shift η versus period of disturbance τ for $Sr_w^2/r_c^2 = 10^{-4}$ and for various values of T/r_c^2.

While the water level fluctuation can be measured in a fairly straightforward manner by using a downhole pressure transducer or a float-type water level recorder, there may be greater difficulties in measuring the pressure disturbance in the aquifer. One approach is to measure downhole pressure in a well shut in by packers so that the effect of well bore storage is eliminated. If only one well is available for the phase analysis, the well can be shut in for a period of time and then left open. If two nearby wells are available, the water level and pressure disturbance can be measured simultaneously by leaving one open and shutting in the other.

Alternatively, one can estimate the pressure disturbance to within a multiplicative constant by determining the tidal dilatation of the aquifer. The dilatation can be (1) measured by an in-situ dilatometer installed near the well or (2) estimated by theoretical calculations based on tidal theory. Obviously, in-situ measurements of tidal dilatation provides the most direct information for phase analysis. Unfortunately, such measurements are rare due to the time and expense needed to set up and operate the necessary equipment. In the absence of dilatation measurements, one can resort to theoretical calculations from solid Earth tide theories, the simplest of which assumes an elastic and radially stratified Earth. A disadvantage of this approach is that the theoretical dilatation may differ from the actual dilatation at a site. *Beaumont and Berger* [1975] and *Berger and Beaumont* [1976] compared the observed and theoretical tidal strains of the M_2 and O_1 constituents at several strainmeter stations in the United States. They found that the areal strains observed at these stations differ in phase from the areal strains calculated from solid Earth tide theory by as much as 12.1°, with an average difference of about 5° [see *Berger and Beaumont*, 1976, p. 1844, Table 6]. By correcting for the effects of ocean tide loading, topography, and lateral inhomogeneities, the largest phase difference could be reduced to 7.2°, but an average difference of about 4° still remained. If we assume that the strain measurements are made on the surface of an isotropic elastic body, then the areal strain ε_A and the dilatation ε_V is related by

$$\varepsilon_V = \frac{1-2\nu}{1-\nu}\varepsilon_A \tag{19}$$

where ν is Poisson's ratio. In other words, the areal strain should be in phase with the dilatation, and the findings of Berger and Beaumont concerning the phase of the areal strain should apply also to dilatation. These findings suggest that considerable care should be exercised in determining the phase of the local dilatation from theoretical Earth tide calculations.

CASE STUDY

The U.S. Geological Survey has been monitoring water levels in a network of wells along the San Andreas and associated fault zones in Southern California. The data collection is part of an effort to study the relationship between anomalous water level fluctuations and tectonic strains along fault zones. We analyze data from an 88-m-deep well at the Gold Hill site, which is approximately 11 km southeast of Parkfield, California. The well is cased from land surface to a depth of 18.3 m, below which it is left open in fractured crystalline rock. The casing and open-hole radius are both 7.0 cm. The well is instrumented to record, at 15-min intervals, the water level in the well, water temperature, barometric pressure, and rainfall. In the present study we analyze water levels and barometric pressures recorded over a 120-day period from February 24 through June 23, 1985.

The Gold Hill site is chosen for study because two Sacks-Everston dilatometers [*Sacks et al.*, 1971] are installed at the site. Typically, the dilatometer is installed in a borehole at a depth between 100 and 200 m. It is cemented in the borehole with expansive grout having density characteristics approximating those of the granite host material. The borehole is then filled to the surface with cement [*Johnston et al.*, 1986]. Dilatation measurements are recorded once every 10 min. Data from one of these dilatometers are provided to us by M. J. S. Johnston of the U. S. Geological Survey. Unfortunately, the dilatation record does not coincide exactly with the water level–barometer record. Dilatation data from January 15 through May 13, 1985, are chosen for analysis. There is a data gap from February 14 through 22, so that the record actually consists of data for 110 days.

The raw data generally require further processing before they can be analyzed. Occasional 3-hour gaps, created by transmission errors, are filled by cubic spline interpolation. High-frequency noise is removed by a smoothing operator that effectively cuts off frequencies higher than about 0.5 cycle/hour. These smoothing operators are described by *Godin* [1972, p. 149] and are summarized in Appendix B. The

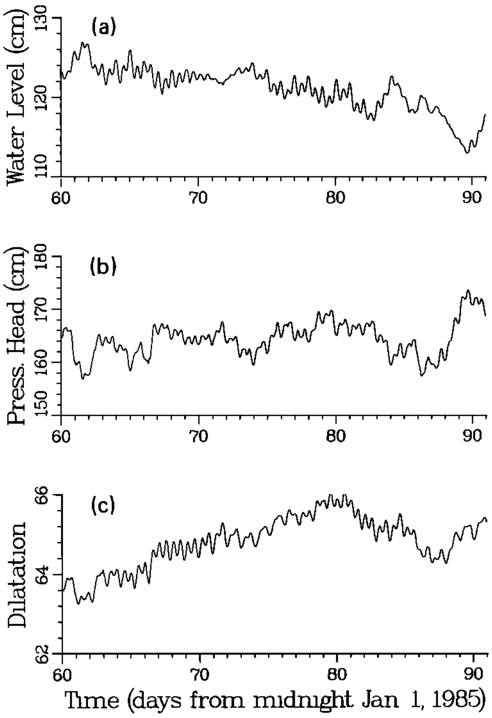

Fig. 5. Plot of smoothed records of (*a*) water level, (*b*) barometric pressure, and (*c*) dilatation at Gold Hill site from March 1 through March 30, 1985. Dilatation is expressed in an uncalibrated unit that is linearly proportional to the actual dilatation.

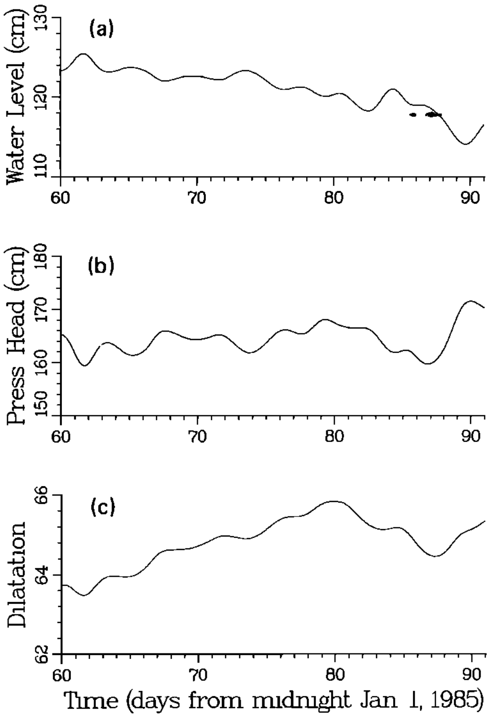

Fig. 6. Plot of low-frequency (<0.8 cycles/day) components of (*a*) water level, (*b*) barometric pressure, and (*c*) dilatation at Gold Hill site from March 1 through March 30, 1985. Dilatation is expressed in an uncalibrated unit that is linearly proportional to the actual dilatation.

smoothed record is decimated to hourly spaced data, which is the standard interval for tidal analysis.

Figure 5 shows the smoothed, hourly spaced record for water level, barometric pressure, and dilatation from March 1 through March 30, 1985. Both the water level and barometric pressure are expressed as pressure head and are referenced to arbitrary datum planes. The dilatation is expressed in uncalibrated units that are lineraly related to strain. As we are interested only in phase relationships, the calibration constant is not required. The semidiurnal and diurnal fluctuations in the barometric pressure are primarily attributed to solar heating of the atmosphere. The corresponding fluctuations in water level and dilatation represent the combined effects of barometric and tidal loading. In addition to semidiurnal and diurnal fluctuations, the barometric pressure also shows larger-amplitude cycles with periods ranging from about 4 days to a week. Such cycles are probably due to movements of weather systems past the site. Their effects can be clearly seen in the water level and dilatation records. The long-term decline of the water level shown in Figure 5*a* is a seasonal trend. On the other hand, the long-term rise of the dilatation shown in Figure 5*c* is believed to reflect the continued curing of the grout rather than a buildup of (compressive) dilatational strain in the aquifer (M. J. S. Johnston, U.S. Geological Survey, oral communication, 1986). Such long-term drifts have also been noticed at other dilatometer sites during the first year after the instruments were installed (see, for example, *Wyatt et al.*, [1983]).

The smoothed, hourly records are further processed by removal of the low-frequency components. This is accomplished by a two-step procedure. First, a low-pass filter (details in Appendix B) is used to isolate frequency components from 0 to about 0.8 cycles/day. The result of this low-pass operation is shown in Figure 6. Second, the low-frequency records are subtracted from the smoothed, hourly records to yield records that contain solely the semidiurnal and diurnal tidal components, along with any noise in this tidal band (see Figure 7). These records are referred to below as the reduced records.

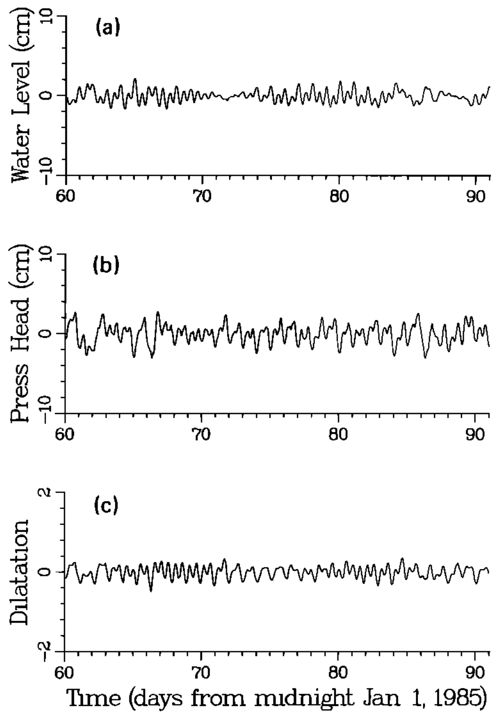

Fig. 7. Plot of reduced records of (*a*) water level, (*b*) barometric pressure, and (*c*) dilatation at Gold Hill site from March 1 through March 30, 1985. Dilatation is expressed in an uncalibrated unit that is linearly proportional to the actual dilatation.

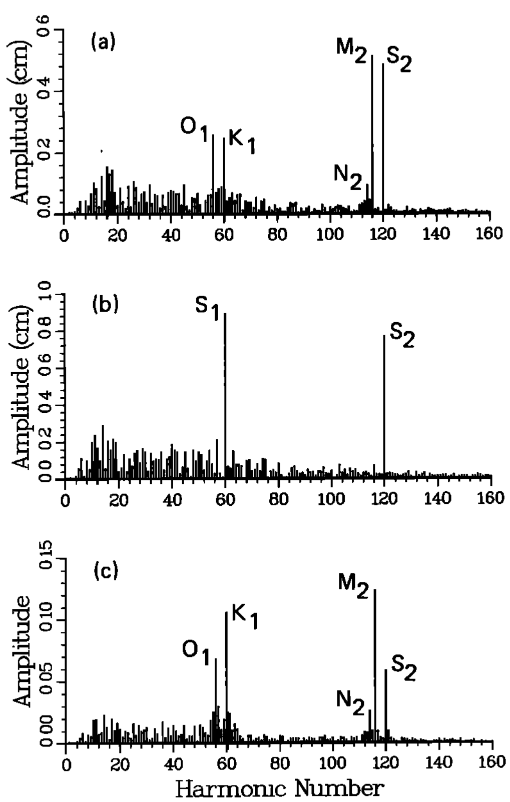

Fig. 8. Amplitude spectra obtained from Fourier transform of reduced records of (*a*) water level, (*b*) barometric pressure, and (*c*) dilatation at Gold Hill site from March 1 through April 29, 1985. Dilatation is expressed in an uncalibrated unit that is linearly proportional to the actual dilatation.

Prior to the phase analysis, Fourier transforms are applied to the reduced records. Figure 8 shows the computed amplitude spectra for a 60-day record from March 1 through April 29, 1985. For the water level and dilatation spectra, one can see the main lines for the O_1, K_1, N_2, M_2, and S_2 constituents. (Actually, the S_2 line also includes the K_2 constituent but the two have periods that are sufficiently close that they cannot be separated with a record length of 60 days.) The spectra for barometric pressure, however, show only two main lines, denoted in Figure 8*b* by S_1 and S_2. These two lines occur at the same harmonic numbers as the K_1 and S_2 lines of the water level and dilatation spectra. This means that the K_1 and S_2 constituents will, in general, represent the combined effects of barometric and tidal loading. To use these two components in the phase analysis, the barometric effects must be removed from the water level and dilatation data.

Existing methods of removing barometric effects [e.g., *Clark*, 1967] is based on the concept of barometric efficiency, which was introduced by *Jacob* [1940] to denote the ratio of a change in water level in a well to the change in barometric pressure. This concept, however, is insufficient for the present study. Since the water level responds dynamically to Earth tide loading, it should respond likewise to barometric loading. In other words, the barometric efficiency should generally be frequency dependent and there will also be a phase shift, with the water level change lagging the barometric pressure change. Rather than developing a new procedure for removing barometric effects, we disregard the K_1 and S_2 constituents in the present study. We also neglect the N_2 constituent; due to its small amplitude, the computed phase will be subject to large errors. We are thus left with the O_1 and M_2 constituents in the phase analysis. Fortunately, these two constituents have large signal-to-noise ratios and they are not contaminated by barometric effects.

While the Fourier transform described above provides the phases of the tidal constituents at various harmonic numbers, we prefer to determine the phases by least squares fitting so that the exact frequencies of the tidal constituents can be used. The reduced water level record is expressed as

$$h(t_j) = \sum_{k=1}^{N} a_k \cos(\omega_k t_j + \zeta_k) + e_j \tag{20}$$

where

- t_j time of data point j, T;
- $h(t_j)$ reduced water level at time t_j, L;
- N number of tidal constituents used in analysis;
- ω_k angular frequency of the kth tidal constituent, T^{-1};
- a_k amplitude of the kth constituent in the reduced water level record, L;
- ζ_k phase angle of the kth constituent in the reduced water level record, degree or radian;
- e_j residual of data point j in the reduced water level record, L.

Similarly, the reduced dilatation record is expressed as

$$\varepsilon_V(t_j) = \sum_{k=1}^{N} A_k \cos(\omega_k t_j + \theta_k) + E_j \tag{21}$$

where $\varepsilon_V(t_j)$ is the reduced dilatation at time t_j, and A_k, θ_k, and E_j take on analogous roles as a_k, ζ_k, and e_j as defined above. The amplitudes a_k and A_k and the phases ζ_k and θ_k are computed by the least squares fitting procedure described by *Bloomfield* [1976, chapter 2]. The phase shift associated with each constituent is computed by

$$\eta_k = \zeta_k - \theta_k \tag{22}$$

In the present study, five tidal constituents (O_1, K_1, N_2, M_2, and S_2) are used ($N = 5$). The angular frequencies of these constituents are well-known (see, for example, *Melchior* [1978]) and are given in Table 1. To obtain a measure of the error in the estimated phases, the water level and dilatation records are each divided into four 30-day segments, and the least squares fitting procedure is applied to each segment. Due to insufficient length in the dilatation data, the third and fourth segments overlap by 10 days; otherwise, all segment are independent of each other. For comparison, the fitting procedure is also applied to the entire data records.

Tables 2 and 3 show the results of the least squares fitting for the O_1 and M_2 phases, which are all reckoned from mid-

TABLE 1. Angular Frequencies and Periods of Five Tidal Constituents Used in Least Squares Fitting

Name of Constituent	Angular Frequency, degree/hour	Period, hour
O_1	13.943	25.819
K_1	15.041	23.934
N_2	28.440	12.658
M_2	28.984	12.421
S_2	30.000	12.000

Data from *Melchior* [1978].

TABLE 2. Results of Least Squares Fitting Showing Phases of O_1 Constituent

Water Level		Dilatation		
Length of Data Record, 1985	Phase of O_1 Constituent	Length of Data Record, 1985	Phase of O_1 Constituent	Phase Shift
		Jan. 15 to Feb. 13	182.9°	
Feb. 24 to March 25	176.1°	Feb. 24 to March 25	189.5°	−13.4°
March 26 to April 24	190.6°	March 26 to April 24	173.9°	16.7°
April 25 to May 24	188.1°	April 14 to May 13	186.7°	1.4°
May 25 to June 23	175.3°			
Mean	182.5°		183.3°	1.6°
Standard deviation	8.0°		6.8°	15.1°
Feb. 24 to June 23	183.6°	Jan. 15 to Feb. 13 and Feb. 24 to May 13	184.9°	−1.3°

night of January 1, 1985. For both water level and dilatation there is considerable variation in the O_1 phases, with a standard deviation of up to 8°. (Due to the few number of data segments, the standard deviation should be regarded only as a qualitative indication of the error in the phase estimates.) As a consequence, there is an even larger variation in the computed phase shift. The mean phase shift of 1.6° is contrary to our expectation that the water level fluctuation should lag behind the dilatation fluctuation. However, the large standard deviation of 15.1° indicates that the mean value is subject to such large errors that it is not meaningful for further analysis. On the other hand, the estimates for the M_2 phase show much less variation. For both water level and dilatation, the standard deviations of the phase angles are less than 1°, and the standard deviation for the phase shift is less than 2°. The M_2 phase shift determined from using the entire record is also very close to the mean value determined from the record segments, which is −11.6°.

Once the phase shift is determined, the value of $T\tau/r_c^2$ can be obtained from Figure 2. To account for uncertainties, we consider the range of phase shifts spanning one standard deviations on each side of the mean. The storage coefficient is assumed to range from 10^{-4} to 10^{-6}. Table 4 shows the range of values of $T\tau/r_c^2$ determined from the M_2 phase shift. Note that $T\tau/r_c^2$ varies from 76 to 156. Since $\tau = 12.421$ hours and $r_c = 7$ cm, the value of T is computed to range from 8×10^{-6} to 2×10^{-5} m²/s.

Although there has been no aquifer test conducted at the Gold Hill well, the transmissivity estimated from the present phase analysis can be compared to the transmissivity determined from the analysis of water level response to an earthquake. On August 4, 1985, the North Kettleman Hills earthquake occurred with the epicenter approximately 40 km from the Gold Hill. At the Gold Hill well, a water level drop of 3.6 cm was observed. After the earthquake, the water level did not recover to the preearthquake level. *Roeloffs and Bredehoeft* [1985] modeled the earthquake as an elastic dislocation, which caused a permanent (extensional) dilatation at the Gold Hill site. Assuming that the earthquake occurred instantaneously, the drop in pressure head in the aquifer at Gold Hill should also have occurred instantaneously. However, the water level in the Gold Hill well took approximately 45 min to drop to its new level. If this gradual decline is attributed to the effects of well bore storage, then the water level data can be analyzed as if they were the response to a slug test [*Cooper et al.*, 1967]. In this case, the response of the well to an instantaneous drop of pressure head in the aquifer can be considered as the response to an instantaneous increase of water level in the well. Using such an approach, Roeloffs and Bredehoeft (unpublished manuscript, 1987) estimated an aquifer transmissivity of 2×10^{-5} m²/s. This value agrees with the upper limit of the transmissivity range estimated by the phase analysis. The general agreement between the two independent estimates supports our assumption that the phase shift is primarily caused by well bore storage effects rather than the orientation of a major fracture or a set of subparallel fractures intersecting the well.

SUMMARY

We can summarize the main points of our study as follows.

1. Due to the effects of a finite well bore, there exists a phase shift between the pressure head disturbance caused by

TABLE 3. Results of Least Squares Fitting Showing Phases of M_2 Constituent

Water Level		Dilatation		
Length of Data Record, 1985	Phase of M_2 Constituent	Length of Data Record, 1985	Phase of M_2 Constituent	Phase Shift
		Jan. 15 to Feb. 13	95.0°	
Feb. 24 to March 25	82.8°	Feb. 24 to March 25	96.6°	−13.8°
March 26 to April 24	84.3°	March 26 to April 24	94.5°	−10.2°
April 25 to May 24	83.9°	April 14 to May 13	94.8°	−10.9°
May 25 to June 23	84.7°			
Mean	83.9°		95.2°	−11.6°
Standard deviation	0.8°		0.9°	1.9°
Feb. 24 to June 23	83.9°	Jan. 15 to Feb. 13 and Feb. 24 to May 13	95.6°	−11.7°

TABLE 4. Range of Values of $T\tau/r_c^2$ Determined From Phase Analysis of M_2 Constituent

Phase Shift	$T\tau/r_c$ Computed for $S = 10^{-4}$	$T\tau/r_c$ Computed for $S = 10^{-5}$	$T\tau/r_c$ Computed for $S = 10^{-6}$
−9.7°	111	134	156
−11.6°	90	109	128
−13.5°	76	92	108

dilatation of a confined aquifer and the water level response in an open well tapping the aquifer. Our theoretical analysis shows that a significant phase shift may still be present even if the disturbance is slowly varying, as in the case of Earth tides.

2. The presence of this phase shift suggests a method for estimating aquifer transmissivity from water level records. This is accomplished by making long-term measurements of water level in a well and pressure disturbance in the aquifer. While the former can be measured by conventional methods, the latter will require downhole pressure measurements in a shutin well or dilatational measurements by means of in situ dilatometers. Theoretical calculations of tidal dilatation should be used with caution because available data suggest that the theoretical and actual dilatations may differ significantly in phase.

3. For the phase analysis, the traditional concept of a (constant) barometric efficiency is insufficient for removal of barometric effects. Development of a method that accounts for the dynamic response of the water level to barometric loading will greatly enhence the phase analysis. If barometric effects are not removed, then the K_1 and S_2 constituents will be contaminated by barometric fluctuations and should not be used in the phase analysis.

4. Once the phase shift is determined, the transmissivity can be estimated by assuming a range of values of the storage coefficient S. Because the phase shift is not very sensitive to S, assuming a relatively large range of S values (several orders of magnitude) can still provide an acceptably narrow range of transmissivity values (within a factor of 2–4).

5. Our methodology is used to analyze water level and dilatation records from the Gold Hill site, near Parkfield, California. For error analysis, the records are divided into 30-day segments. Five tidal constituents are fitted to each segment by least squares. The phase shift computed for the O_1 constituent exhibits such large variations that it is deemed useless for estimating transmissivity. The phase shift of the M_2 constituent shows much less variation and has a mean value of −11.6°. Assuming that the storage coefficient of the aquifer is in the range from 10^{-4} to 10^{-6}, the range of estimated transmissivity is from 8×10^{-6} to 2×10^{-5} m²/s. The value at the upper limit agrees with the transmissivity estimated by other investigators from analysis of water level response to an earthquake.

APPENDIX A: SOLUTION FOR DRAWDOWN IN A WELL WITH PERIODIC DISCHARGE

In radial coordinates, the equation governing fluid flow in a homogeneous, isotropic, confined aquifer is

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} - \frac{S}{T}\frac{\partial s}{\partial t} = 0 \tag{A1}$$

The boundary condition for periodic discharge at a volumetric rate $Q_0 \exp(i\omega t)$ from the aquifer into the well is

$$2\pi r_w T\left(\frac{\partial s}{\partial r}\right)_{r=r_w} = -Q_0 \exp(i\omega t) \tag{A2}$$

We also require that

$$s \rightarrow 0 \qquad r \rightarrow \infty \tag{A3}$$

The steady state solution will be of the form

$$s(r, t) = G(r) \exp(i\omega t) \tag{A4}$$

where $G(r)$ is a (complex) function of r only. Substituting (A4) into (A1), (A2), and (A3) yields

$$\frac{d^2G}{dr^2} + \frac{1}{r}\frac{dG}{dr} - \frac{i\omega S}{T}G = 0 \tag{A5}$$

$$2\pi r_w T\left(\frac{dG}{dr}\right)_{r=r_w} = -Q_0 \tag{A6}$$

$$G \rightarrow 0 \qquad r \rightarrow \infty \tag{A7}$$

The solution to (A5) satisfying (A7) is

$$G = CK_0(i^{1/2}\alpha) \tag{A8}$$

where C is a constant to be determined,

$$\alpha = \left(\frac{\omega S}{T}\right)^{1/2} r$$

and $K_0(i^{1/2}\alpha)$ is the modified Bessel function of the second kind, of order zero. Equation (A8) can be written in terms of the Kelvin functions of order zero, Ker (α) and Kei (α), as

$$G = C[\mathrm{Ker}(\alpha) + i\,\mathrm{Kei}(\alpha)] \tag{A9}$$

To determine C, we substitute (A9) into (A6). Using the formulae [*Abramowitz and Stegun*, 1964, p. 380, equation 9.9.17]

$$\frac{d}{dz}\mathrm{Ker}(z) = \frac{1}{2^{1/2}}[\mathrm{Ker}_1(z) + \mathrm{Kei}_1(z)] \tag{A10}$$

$$\frac{d}{dz}\mathrm{Kei}(z) = -\frac{1}{2^{1/2}}[\mathrm{Ker}_1(z) - \mathrm{Kei}_1(z)] \tag{A11}$$

where $\mathrm{Ker}_1(z)$ and $\mathrm{Kei}_1(z)$ are the Kelvin functions of order one, we obtain

$$C = \frac{Q_0}{2\pi T}[\Phi + i\Psi] \tag{A12}$$

where Φ and Ψ are defined by (8) and (9), respectively.

Substituting (A12) and (A9) into (A4) yields the drawdown in the aquifer

$$s(r, t) = \frac{Q_0}{2\pi T}\{[\Phi\,\mathrm{Ker}(\alpha) - \Psi\,\mathrm{Kei}(\alpha)] + i[\Psi\,\mathrm{Ker}(\alpha) + \Phi\,\mathrm{Kei}(\alpha)]\}\exp(i\omega t) \tag{A13}$$

The drawdown in the well s_w can be obtained from (A13) by replacing r by r_w and α by α_w, which is defined by (10).

APPENDIX B: SUMMARY OF SMOOTHING AND LOW-PASS FILTERS USED FOR DATA REDUCTION

The smoothing and low-pass filters are described in detail by *Godin* [1972]. The basic operation is the consecutive summation of n data points and is denoted by $\mathscr{A}_n$ [*Godin*, 1972, p. 62]. Thus $\mathscr{A}_n/n$ denotes the average of n consecutive data point. The average value is assigned to the center of the interval over which the n data points are averaged. For example, consider the data sequence $z_1, z_2, z_3, \cdots, z_M$. The operation $\mathscr{A}_n/n$ consists of calculating

$$X_k = \sum_{j=0}^{n-1} z_{j+k} \qquad k = 1, 2, 3, \cdots, M - n + 1 \tag{B1}$$

and results in the sequence $X_1, X_2, \cdots, X_{M-n+1}$. If n is odd, X_k is assigned to the time of data point $z_{k+(n-1)/2}$. If n is even, X_k is assigned to the time halfway between data points $z_{k+n/2-1}$ and $z_{k+n/2}$.

Both the smoothing and low-pass filters are of the form $\mathscr{A}_n{}^2\mathscr{A}_{n+1}/[n^2(n+1)]$, which requires three averaging operations [c.f. *Godin*, 1972, p. 66]. The $\mathscr{A}_6{}^2\mathscr{A}_7/(6^2 \times 7)$ operator is used to smooth data recorded at 10-min intervals, while the $\mathscr{A}_4{}^2\mathscr{A}_5/(4^2 \times 5)$ operator is used to smooth data recorded at 15-min intervals [*Godin*, 1972, p. 149]. After smoothing, only hourly data are kept. The $\mathscr{A}_{24}{}^2\mathscr{A}_{25}/(24^2 \times 25)$ operator is used as a low-pass filter to isolate the low-frequency components.

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